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# Fusion arrest and collapse phenomena due to Kerr-nonlinearity in quadratic media

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**Abstract:** Emphasizing collapse phenomena it is investigated to what extend the always present cubic nonlinearity affects the properties of soliton interaction in quadratic bulk media. An effective particle approach is applied and verified by numerical simulations.

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The understanding of how and to what extend the cubic nonlinearity affects beam propagation and spatial soliton formation in quadratic media is of vital importance not only in fundamental nonlinear physics but also in the applied field of rewritable optical circuitry, i.e. switching devices, junctions etc.[1, 2].

All quadratic materials have an inherent cubic nonlinearity that becomes important at high intensities or when the fundamental wave (FW) and its second harmonic (SH) do not meet the phase-matching condition. Recently also very strong induced cubic nonlinearities have been achieved via quasi-phase-matching techniques[3, 4].

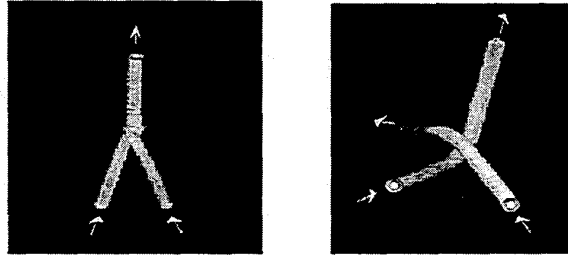


Fig. 1. Fusion and spiraling. Examples of spatial soliton interactions in bulk media[5].

We consider beam propagation under type-I SHG conditions in lossless bulk  $\chi^{(2)}$  materials with a non-vanishing  $\chi^{(3)}$  nonlinearity. It is known that in pure  $\chi^{(2)}$  systems a single soliton can never collapse[6] whereas in systems with both nonlinearities stable single soliton propagation can only be achieved for small effective  $\chi^{(3)}$  values and low powers[7]. The well-known system of normalized nonlinear equations describing the propagation of the slowly varying envelopes are[8]

$$i \frac{dA_1}{dz} + \frac{1}{2} \nabla_{\perp}^2 A_1 + A_2 A_1^* + \gamma \left( \frac{1}{2} |A_1|^2 + |A_2|^2 \right) A_1 = 0 \quad (1)$$

$$i \frac{dA_2}{dz} + \frac{1}{4} \nabla_{\perp}^2 A_2 - \beta A_2 + A_1^2 + \gamma \left( |A_2|^2 + 2|A_1|^2 \right) A_2 = 0 \quad (2)$$

where  $\beta$  is the effective phase-mismatch parameter and  $\gamma$  determines the relative strength of the effective  $\chi^{(3)}$  nonlinearity to that of the effective  $\chi^{(2)}$  nonlinearity.  $A_1$  and  $A_2$  are, respectively, the FW and the SH. The system can be derived from a Lagrangian density. In the case of two interacting solitons

$$A_1 = A_1^{(1)} + A_1^{(2)}, \quad A_2 = A_2^{(1)} + A_2^{(2)} \quad (3)$$

the extra terms in the Lagrangian density, containing contributions from both solitons, can be treated as first order perturbations. The solitons are each characterized by three parameters ( $\lambda$  defining the soliton profile and  $\nu_x$  and  $\nu_y$  accounting for transverse velocities) which are then allowed slow adiabatic variation. The first order correction to the Lagrangian describes how the soliton parameters change during propagation. Through the effective particle approach the dimensionality of the system can be reduced yielding the following first order correction to the Lagrangian in case of initially identical solitons

$$L = \frac{1}{2} M_R \dot{R}^2 - \frac{1}{2} M_\phi \dot{\phi}^2 - U_{eff} \quad (4)$$

where

$$M_R = \pi \int_0^\infty [|V_s|^2 + 2|W_s|^2] r dr, \quad M_\phi = 2 \frac{\partial M_R}{\partial \lambda} \quad (5)$$

$R$  and  $\phi$  are, respectively, the relative distance between soliton centers and the relative phase between the solitons. The effective potential  $U_{eff}$  consists of three terms: the classical centrifugal potential barrier and an interaction integral for each of the nonlinearities consisting of various coupling terms between the two solitons.

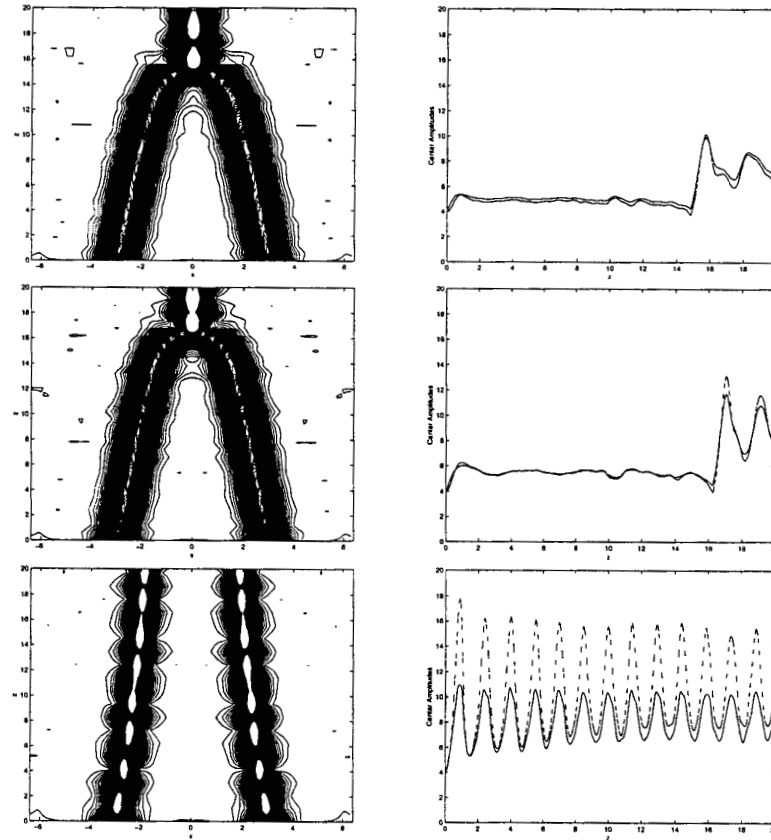


Fig. 2. Fusion arrest due to  $\chi^{(3)}$  nonlinearity. In the contour plot only the FW's are shown. Top:  $\gamma = 0$ , Middle:  $\gamma = 0.01$ , Bottom:  $\gamma = 0.05$ .

The model is based on the theory formulated in [5] for the pure  $\chi^{(2)}$  case. The introduction of a  $\chi^{(3)}$  nonlinearity gives rise to qualitatively different behavior for the different scenarios, i.e. fusion, repulsion and spiraling. As an example we mention the case of fusion where we have observed that the introduction of the  $\chi^{(3)}$  nonlinearity leads to an arrest of the fusion process. This is shown in figure 2 where two identical solitons with no relative phase difference collide.

The collapse effect introduced by the  $\chi^{(3)}$  nonlinearity may lead to other qualitatively new behaviors of the system. The system is investigated analytically under a Gaussian approximation in the effective particle approach and the results are verified numerically.

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